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Brane world with bulk horizons

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Abstract: A brane world in the presence of a bulk black hole is constructed. The brane tension is fine tuned in terms of the black hole mass and cosmological constant. Gravitational perturbations localized on the brane world are discussed.

Keywords: p-branes, Black Holes.

Contents

1. Introduction

A brane world with induced four dimensional gravity was first introduced in [1, 2] on the basis of a AdS_5 bulk geometry. In this scheme normalizable gravitational zero modes are allowed due to the ultraviolet cutoff induced by the brane wall. The dilaton field is constant and the holographic degrees of freedom on the wall define a conformal field theory coupled to gravity [3]. In a series of recent papers [4]–[8] this framework was extended to the non conformal case i.e to dilatonic domain walls. In these cases, both with vanishing and non vanishing cosmological constant, we observe the phenomena of induced four dimensional gravity, however a naked curvature singularity is induced by the non constant dilaton in the bulk at a finite proper distance from the brane wall. The physics interpretation of such a singularity from the four dimensional point of view is still an open problem.

In this letter we will look for a Randall-Sundrum scenario but this time in the bulk geometry of a real black hole with the singularity inside a trapped surface. A similar analysis was with a non static ansatz was first by [9, 10] in the framework of cosmological models. We will work out the static case in a Schwarzschild-AdS bulk metric. This will correspond to a brane world in a thermal bath at the Hawking temperature. The first question we will address would be the fine tuning relations between the brane wall tension and the parameters Λ and M characterizing the Schwarzschild-AdS metric. These fine tuning relations would be obtained by solving the corresponding jump equations once we introduce the wall as an ultraviolet cutoff analogously to the AdS case. Since our space is asymptotically AdS this cutoff could be enough to induce four dimensional gravity on the wall in terms of normalizable graviton zero modes.

2. Construction of the solution

Our starting point is the following five-dimensional action of gravity in the presence of a cosmological constant Λ with a domain wall source term given by:

$$S = \frac{1}{\kappa}\int d^4x dy\sqrt{|g|}[\mathcal{R}-\Lambda] + \int d^4x\sqrt{|\tilde{g}|}V_0\,, \tag{2.1}$$

where V_0 is the tension of the brane and $\tilde{g}_{mn} = g_{\mu\nu}\delta^\mu_m\delta^\nu_n$ the induced metric on the brane.

We are interested in a solution resembling the Schwarzschild-AdS solution, which is given by [11]:

$$ds^2 = \left(1 + R^{-2}r^2 - \frac{2M}{r^2}\right)dt^2 - \frac{1}{(1 + R^{-2}r^2 - 2M/r^2)}dr^2 - r^2 d\Omega_3^2\,. \tag{2.2}$$

where $R = 12\Lambda^{-1}$ and M is basically the black hole mass. In order to apply the Randall-Sundrum program to this type of metrics, we consider a new set of coordinates defined by,

$$dz = \frac{1}{\sqrt{1 + R^{-2}r^2 - 2M/r^2}}dr\,, \tag{2.3}$$

therefore ending up in a holographic-like frame

$$ds^2 = A^2(z)dt^2 + B^2(z)d\Omega_3^2 - dz^2\,, \tag{2.4}$$

where $A(z)$ and $B(z)$ are function of the holografic coordinate z, implicitly given by

$$A(r) = \sqrt{1 + R^{-2}r^2 - \frac{2M}{r^2}}\,, \qquad B(r) = r\,, \tag{2.5}$$

with r a function of z, defined by the relation (2.3). Using this ansatz (2.4) in the corresponding equations of motion of (2.1), we get the following system of equations:

$$\begin{aligned} B^{-2}\Lambda - B^{-2}(B')^2 - A^{-1}A'B^{-1}B' &= 0\,,\\ -B^{-2} + B^{-1}B'' + B^{-2}(B')^2 + \Lambda + \frac{1}{6}\kappa V_0\delta(z) &= 0\,,\\ -1 + \frac{1}{2}B^2\Lambda + A^{-1}B^2A'' + 2BB'' + (B')^2 + 2A^{-1}A'BB' + \frac{1}{2}\kappa V_0\delta(z) &= 0\,, \end{aligned} \tag{2.6}$$

where " $'$ " means derivatives with respect to z and have used the fact that $d\Omega_3^2$ is a three-dimensional sphere of radius one.

To obtain the desired solution, we use the fact that (2.5) is a solution of the action without source term, therefore in our case the solution to the full equations (2.6) is given by the functions (2.5) with a modification of the relation between z and r, defined as follows

$$|\bar{z}| = z_0 - \frac{R}{2}\log\left|\frac{2R^{-1}rA + 2R^{-2}r^2 + 1}{\sqrt{1 + 8MR^{-2}}}\right|\,, \tag{2.7}$$

where $\bar{z} = z_0 - z$, which translates into:

$$d|\bar{z}| = -\frac{1}{\sqrt{1 + R^{-2}r^2 - 2M/r^2}} dr\,. \tag{2.8}$$

The modulus of the radial coordinate $\bar{z}$ runs between the the position of the brane at $\bar{z} = 0$ and the black hole horizons at $|\bar{z}| = z_0$. In this coordinates this means that the brane is located somewhere between the horizon and the boundary.

By solving the jump equations we find that the brane tension V_0 is given in terms of the cosmological constant Λ, the black hole mass M and $r(\bar{z} = 0)$, provided the brane is located at the origin in $\bar{z}$ coordinates, by the following relation:

$$V_0 = \frac{6}{\kappa r(0)} \sqrt{1 + R^{-2}r(0)^2 - \frac{2M}{r(0)^2}}\,. \tag{2.9}$$

If the black hole horizon is smaller than the AdS radius $M < R^2$, we could choose to do the cutoff at $r(\bar{z} = 0) = R$ and the above formula reduces to:

$$V_0 = \frac{6}{\kappa R^2}\sqrt{2R^2 - 2M} = -\frac{1}{\kappa}\sqrt{-\frac{\Lambda}{6}\left(1 - \frac{1}{12}M\Lambda\right)}\,. \tag{2.10}$$

Note that in the limit $M \to 0$, we recover the Randall-Sundrum relation between V_0 and Λ [1, 2].[1]

In summary what we have done is basically to consider Schwarzschild-AdS space time with a brane located at a given distance $r(0)$ from the event horizon. Then replace the part of the space time outside the brane ($r > r(0)$) with a copy of the inner part, ending up with a finite range for the radial variable. It is important to note that this space time comes with two space-like singularities hidden inside the event horizons. Comparison with the dilatonic solution found on previous work [7], shows that the role of the singularity on those solutions is replaced by the event horizon in this new model. Nevertheless we also have a non isotropic worldbrane, the time direction scales differently than the space directions under radial flow.

3. Gravitational perturbations

To calculate the behavior of the graviton we add small fluctuations $h_{\mu\nu}$ to the above background, choosing the following gauge:

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + h_{\alpha\beta}\delta^\alpha_\mu \delta^\beta_\nu\,, \tag{3.1}$$

where α, β run over the coordinates t and the angular coordinates x^m. Furthermore we have $h = g^{\mu\nu}h_{\mu\nu} = A^{-2}h_{tt} + B^{-2}\bar{h}$ and $\nabla h = 0$, $\bar{\nabla}^m h_{m\mu} = 0$, where $\bar{\nabla}_m$ stands

[1] Also the limit $R \to \infty$ is well defined, recovering the standard Schwarzschild solution in Minkowski space, where the relation between r and $\bar{z}$ is: $\bar{z} = z_0 - \sqrt{r^2 - 2M}$.

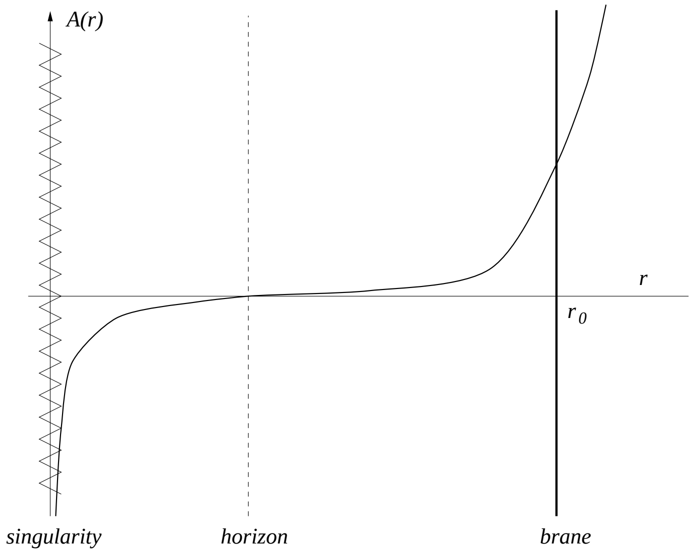

Figure 1: Graviton profile on the Schwarzschild-AdS space-time. For large r the graviton behaves like in ordinary AdS space and is not normalizable. A cutoff in form of a brane is needed at $r = r_0$. For small values of r, the graviton also diverges, but is hidden behind a horizon.

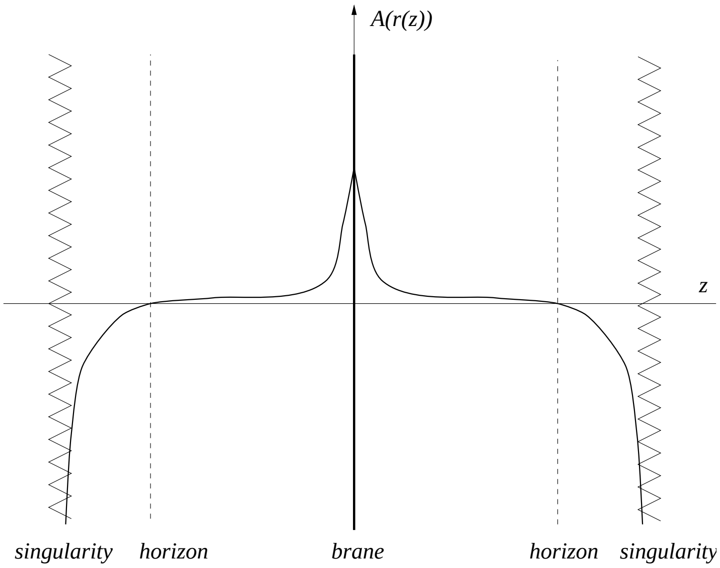

Figure 2: The profile of normalized graviton after the cut off. The thrown away part is replaced by a copy of the space with $z < z_0$. The graviton is localized around the brane. The space-time ends in two singularities which do not harm the causal structure of the brane world, since they are hidden by event horizons.

for the covariant derivative of the angular coordinates. Notice that this is not the usual de Donder gauge since the perturbation is not traceless. Nevertheless if we are interested in a real graviton with two helicity states more constraints should be added.

The equation of motion, on this gauge for the fluctuation $h_{\mu\nu}$ is:

$$\nabla^2 h_{\mu\nu} - 2\nabla^\rho \nabla_{(\mu} h_{\nu)\rho} + \frac{1}{3}\Lambda h_{\mu\nu} - \frac{1}{2}\Lambda g_{\mu\nu} h = 0\,, \tag{3.2}$$

Introducing the background (2.4), the components $\{tt\}$ and $\{zz\}$ of the above equation, reduce to:

$$\begin{aligned} & A^{-3}A'\partial_z h_{tt} - 2A^{-4}(A')^2 h_{tt} + B^{-3}B'\partial_z \bar{h} - 2B^{-4}(B')^2 \bar{h} = 0 \\ & \partial_z^2 h_{tt} + A^{-2}\partial_t^2 h_{tt} - B^{-2}\bar{\nabla}^2 h_{tt} 3(B^{-1}B' - A^{-1}A')\partial_z h_{tt} + \\ & \qquad + \left(4A^{-2}(A')^2 - \frac{1}{3}\Lambda\right) h_{tt} + \frac{1}{2}A^2 h = 0 \end{aligned} \tag{3.3}$$

To describe four-dimensional zero modes, we consider eigenfunction of the world brane variables x^α, satisfying

$$A^{-2}\partial_t^2 h_{tt} - B^{-2}\bar{\nabla}^2 h_{tt} = 0\,. \tag{3.4}$$

Under these conditions we find a very simple solution:

$$h_{tt} = A^2\,, \qquad \bar{h} = B^2\,. \tag{3.5}$$

In principle we could turn on more degrees of freedom, to determine a more realistic graviton, nevertheless this mode shows the correct behavior to illustrate the location of the perturbation, and its normalizability. Note that the specific form of our perturbation reproduces the desired localization on the brane (see figures 1 and 2) as well as the normalizability condition.

To end this letter we would like to relate the world brane Newton constant with the five dimensional Newton constant. To proceed on this direction we note that a straightforward definition is not possible since the obvious Kaluza-Klein reduction gives no terms in the effective action that could be related to the Einstein term. This is a consequence of the anisotropy of the world brane. Fortunately far from the event horizon this space time looks like AdS, therefore our brane becomes isotropic with warp factor A^2. Then we can proceed as usual to define the Newton constant.

The Newton constant, far from the horizon is essentially AdS in static coordinates plus a correction coming from the black hole:

$$M_4^2 = M_5^3 \int_0^{z_0} dz A^2(r(\bar{z})) = r_0 \left(1 + \frac{r_0^2}{R^2} + \frac{2M}{r_0^2}\right) \tag{3.6}$$

For $r_0 = R$ and $M \ll R$

$$M_4^2 = M_5^3 \sqrt{\frac{-64}{3\Lambda}} \left(1 - \frac{1}{8}M\Lambda\right) \tag{3.7}$$

Again the warp factor defines the hierarchy and goes like A^2.

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